

B.Sc. Physics Notes

Subject Physics

Paper B

Chapter 3 Gauss's Law

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Topics

1. Electric field due to infinite line of charge using Gauss's Law.
2. Electric field intensity due to spherical charged shell of constant surface charged density using Gauss's Law.
3. Calculate charge of an isolatar conductor using Gauss's Law.

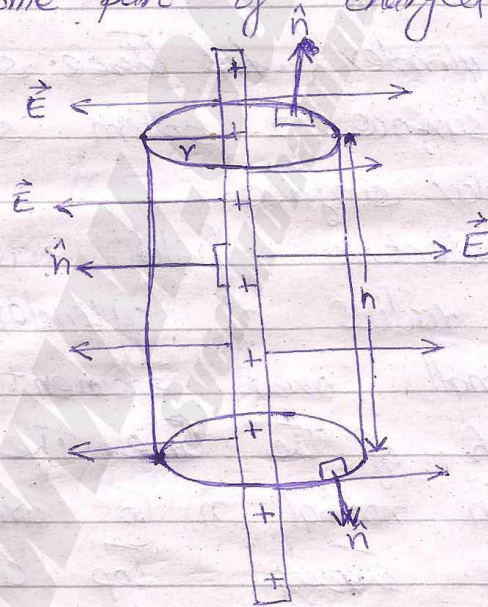
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Q Calculate electric field due to infinite line of charge using Gauss's Law.

Ans Electric Field Due to Infinite Line of Charge:-

Consider a straight positively charged wire having infinite length and negligible thickness. Now take a cylinder called gaussian surface having radius r and height h which encloses the some part of charged wire.



The P is the point on the surface of the cylinder

where we want to calculate electric field. Take a small length element dz on the line having charge dq . The linear charge density λ is given

$$\lambda = \frac{dq}{dz}$$

$$dq = \lambda dz$$

The net charge enclosed by the cylinder having height h is

$$q = \lambda \int dz$$

$$q = \lambda h \quad \text{--- (i)}$$

The cylinder has three surfaces

1. Top surface
2. Bottom surface
3. Circular surface

The electric field will be along radius of cylinder. Take a small area element da of each of the line. The angle between normal \vec{n} to area element through top circular surface and bottom circular surface is 90° . So the electric flux will be zero.

The electric flux angle between normal to area element through curved surface is 0° . Thus the electric flux will be

$$\begin{aligned}\phi_{\text{curved}} &= \int \vec{E} \cdot d\vec{a} \\ &= \int E da \cos \theta \\ &= \int E da \cos 0^\circ \\ &= \int E da\end{aligned}$$

$$\phi_{\text{curved}} = E(2\pi r h)$$

The net flux will be

$$\begin{aligned}\phi_e &= \phi_{\text{top}} + \phi_{\text{bottom}} + \phi_{\text{curved}} \\ &= 0 + 0 + E(2\pi r h)\end{aligned}$$

$$\phi_e = E(2\pi r h) \quad \text{--- (2)}$$

Put eq (1) and eq (2) in Gauss's Law

$$\phi_e = \frac{q}{\epsilon_0}$$

$$E(2\pi r h) = \frac{\lambda h}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

In vector form

$$\vec{E} = \frac{\lambda}{2\pi \epsilon_0 r} \hat{r}$$

Q) State Gauss's law and apply it to calculate electric field intensity due to spherical charged shell of constant surface charge density.

Ans Gauss's Law

The Gauss's law states that the electric flux passing through any closed surface is equal to $\frac{1}{\epsilon_0}$ times charge enclosed by that surface.

Shell Theorems

There are two shell theorems established by using Gauss's law for uniform spherical shell of charge having constant surface charge density and stated as.

First Shell Theorem

A uniform spherical shell of charge behave for external points in such a way that all its charge was

concentrated at its center.

Proof

Consider a truly charged spherical shell of radius R such as charge is sprayed uniformly over the surface of balloon having radius R . Take a small area element da of shell having charge dq .

The surface charge density σ is defined as

$$\sigma = \frac{dq}{da}$$

$$dq = \sigma da$$

$$\int dq = \int \sigma da$$

$$q = \sigma(4\pi R^2) \quad \text{--- (1)}$$

Consider a point P outside the shell having distance r from the center of sphere. Imagine a gaussian surface such as point P lies on its surface. Now take a small area element da of gaussian surface shell having

radius r . The angle between normal \vec{n} to this area element and electric field \vec{E} is zero.

Electric flux through area element da is

$$\begin{aligned}\phi_e &= \int \vec{E} \cdot d\vec{a} \\ &= \int E da \cos \theta \\ &= \int E da \cos 0^\circ \\ &= \int E da \\ \phi_e &= E (4\pi r^2) \quad \text{--- (2)}\end{aligned}$$

Put eq (1) and (2) in Gauss' Law.

$$\phi_e = \frac{q}{\epsilon_0}$$

~~$$E(4\pi r^2) = \frac{\sigma(4\pi R^2)}{\epsilon_0}$$~~

~~$$E = \frac{\sigma R^2}{\epsilon_0 r^2}$$~~

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{(4\pi r^2)\epsilon_0}$$

$$= \frac{q}{(4\pi\epsilon_0 r^2)}$$

$$E = \frac{kq}{r^2}$$

This is electric field at point P due to point charge q .

Second Shell Theorem:-

A uniform charged spherical shell exerts no electrostatic force on charged particle placed inside the shell.

Proof

Consider a ~~truly~~ uniformly charged spherical shell having radius R and charge density σ .

The net charge on shell is q .

Take a point P inside the shell where we want to calculate electric field and distance r from center of spherical charged shell. Imagine a gaussian surface such that point P lies on its surface. Now take small area element da of this gaussian shell having radius r .

The angle between normal and \vec{E} is 0° . The electric flux will be

$$\begin{aligned}\phi_e &= \int \vec{E} \cdot d\vec{a} \\ &= \int E da \cos 0 \\ &= \int E da \cos 0^\circ \\ &= \int E da \\ &= E(4\pi r^2) \quad \text{--- ①}\end{aligned}$$

Put eq ① in Gauss's law

$$\phi_e = \frac{q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

Inside the gaussian surface $q=0$

$$E = 0$$

Thus $F = qE = 0$

isolated conductor. The electric field will exert force on electrons and current is established in conductor, but it is experimentally proved that isolated conductor has no current.

If electric field inside isolated charged conductor is zero. The value of electric flux is

$$\phi_e = EA$$

$$\phi_e = 0$$

It means by Gauss's Law flux will be

$$\phi_e = \frac{q}{\epsilon_0}$$

$$0 = \frac{q}{\epsilon_0}$$

It means net charge $q=0$