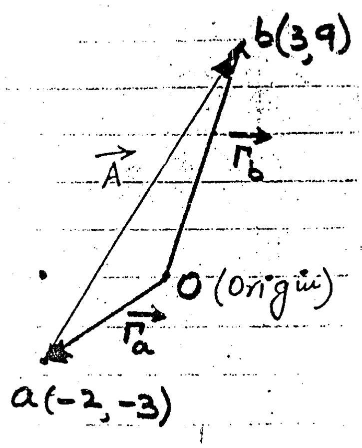
P. 2.1 :- Suppose, in a rectangular coordinate system, a vector $\vec{A}$ has its tail at the point a(-2, -3) and its tip at b(3, 9). Express $\vec{A}$ in terms of a and b. Is this the same as the vector $\vec{P}(5,12)$? Determine the distance between these two points.

Solution :-



Given points are a(-2,-3) and b(3,9)

Coordinates of given vector $\vec{P}(5,12)$ then

(a) Position vector of point a(-2,-3)

$$\vec{r_a} = -2\hat{i} - 3\hat{j}$$

and Position vector of point b(3,9)

$$\vec{r_b} = 3\hat{i} + 9\hat{j}$$

It can be seen from figure by head-to-tail rule

$$\vec{r_a} + \vec{A} = \vec{r_b}$$

$$\therefore \quad \vec{A} = \vec{r_b} - \vec{r_a}$$
$$= (3\hat{i} + 9\hat{j}) - (-2\hat{i} - 3\hat{j})$$
$$= 3\hat{i} + 9\hat{j} + 2\hat{i} + 3\hat{j}$$
$$\vec{A} = 5\hat{i} + 12\hat{j}$$

So. vector $\vec{A}$ has co-ordinates (5,12)
and vector $\vec{P}$ has co-ordinates (5,12) Answer.
$\therefore$ They are same.

(b) - As we have $\vec{A} = 5\hat{i} + 12\hat{j}$

The magnitude of vector $\vec{A}$ determines the distance between points 'a' and 'b'.

$$|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2}$$

$$A = \sqrt{(5)^2 + (12)^2}$$
$$= \sqrt{169}$$
$$\boxed{A = 13 \text{ units}}$$ Answer

P. 2.2 :- A certain corrior of a room is selected as the origin of a rectangular coordinate system. If an insect is crawling on an adjacent wall at a point having coordinates (2, 1) where the units are in metres, what is distance of the insect from this cornor of the room?

Solution :

Cornor of the room is selected as the origin of rectangular co-ordinate system.

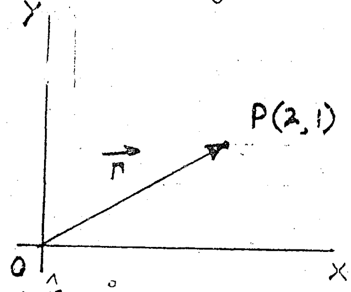
The insect is at point $P(2,1)$

The position vector of insect is

$$\vec{r} = 2\hat{i} + \hat{j} \quad \text{meters.}$$

Distance of the insect from Cornor of the room $= r = ?$

As we have magnitude of the position vector $\vec{r} = x\hat{i} + y\hat{j}$ is



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(2)^2 + (1)^2} \quad \text{meters.}$$

$$r = \sqrt{5} = 2.24\ m \quad \text{Answer.}$$

P. 2.3 :- What is the unit vector in the direction of the vector $\vec{A} = 4\hat{i} + 3\hat{j}$?

Solution :- The given vector is

$$\vec{A} = 4\hat{i} + 3\hat{j}$$

Unit vector $= \hat{A} = ?$

As we have a formula

$$\vec{A} = |\vec{A}|\, \hat{A}$$

or

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} \quad \text{——— (1)}$$

The magnitude of vector $\vec{A} = |\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$$= \sqrt{(4)^2 + (3)^2}$$

$$|\vec{A}| = 5$$

Putting values of $\vec{A}$ and $|\vec{A}|$ in eq (1) we get

$$\boxed{\hat{A} = \frac{4\hat{i} + 3\hat{j}}{5}} \quad \text{Answer.}$$

2.4 :- Two particles are located at $\vec{r_1} = 3\hat{i} + 7\hat{j}$ and $\vec{r_2} = -2\hat{i} + 3\hat{j}$ respectively. Find both the magnitudes of the vector and its orientation w.r.to x-axis.

Solution :- As

$$\vec{r_1} = 3\hat{i} + 7\hat{j}$$

$$\vec{r_2} = -2\hat{i} + 3\hat{j}$$

Magnitude of vector $\vec{r} = ?$

Its orientation w.r.to x-axis $= ?$
(Coordinates)

The vector
$$\vec{r} = \vec{r_2} - \vec{r_1}$$
$$= (-2\hat{i} + 3\hat{j}) - (3\hat{i} + 7\hat{j})$$
$$= -2\hat{i} - 3\hat{i} + 3\hat{j} - 7\hat{j}$$
$$\vec{r} = -5\hat{i} - 4\hat{j}$$

The magnitude of vector $\vec{r}$ can be determined by the formula
$$|\vec{r}| = r = \sqrt{x^2 + y^2}$$

Here $x = -5$ and $y = -4$

$$\therefore \quad r = \sqrt{(-5)^2 + (-4)^2}$$
$$= \sqrt{25 + 16}$$
$$\boxed{r = 6.4}$$ Answer

Now orientation of vector $\vec{r}$ w.r.to x-axis will be its coordinates.

As $\vec{r} = -5\hat{i} - 4\hat{j}$

$\therefore$ Orientation is $(-5, -4)$ with respect to x-axis

Answer.

P. 2.5 :- If a vector $\vec{B}$ is added to vector $\vec{A}$, the result is $6\hat{i} + \hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$, the result is $-4\hat{i} + 7\hat{j}$. What is the magnitude of $\vec{A}$?

Solution :- As given

$$\vec{A} + \vec{B} = 6\hat{i} + \hat{j} \quad \text{——— (1)}$$
$$\vec{A} - \vec{B} = -4\hat{i} + 7\hat{j} \quad \text{——— (2)}$$
$$|\vec{A}| = ?$$

Adding eq (1) and (2)
$$(\vec{A} + \vec{B}) + (\vec{A} - \vec{B}) = (6\hat{i} + \hat{j}) + (-4\hat{i} + 7\hat{j})$$
$$2\vec{A} = 2\hat{i} + 8\hat{j}$$

$A' = \frac{2}{2}\hat{i} + \frac{8}{2}\hat{j} = \hat{i} + 4\hat{j}$

The magnitude of $\vec{A} = |\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$A = \sqrt{(1)^2 + (4)^2} = \sqrt{17}$

$\boxed{A = 4.1}$ Answer.

P. 2.6 :- Given that $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} - 4\hat{j}$ find the magnitude and direction of
(a) $\vec{C} = \vec{A} + \vec{B}$ and (b) $\vec{D} = 3\vec{A} - 2\vec{B}$

Solution :- We have

$$\vec{A} = 2\hat{i} + 3\hat{j}$$
$$\vec{B} = 3\hat{i} - 4\hat{j}$$

(a)
$$\vec{C} = \vec{A} + \vec{B}$$
$$\vec{C} = (2\hat{i} + 3\hat{j}) + (3\hat{i} - 4\hat{j})$$
$$\boxed{\vec{C} = 5\hat{i} - \hat{j}}$$

The magnitude

$$|\vec{C}| = \sqrt{(5)^2 + (-1)^2} = \sqrt{25+1}$$
$$\boxed{|\vec{C}| = 5.09}$$ Answer

Direction

As $\vec{C} = |\vec{C}|\hat{C}$

and $\hat{C} = \frac{\vec{C}}{|\vec{C}|}$

$$\boxed{\hat{C} = \frac{5\hat{i} - \hat{j}}{5.09}}$$ Answer.

(b)
$$\vec{D} = 3\vec{A} - 2\vec{B}$$

Putting values of $\vec{A}$ and $\vec{B}$

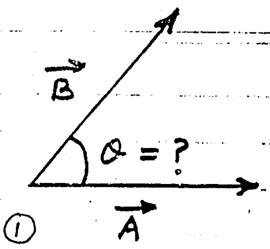
$$\vec{D} = 3(2\hat{i} + 3\hat{j}) - 2(3\hat{i} - 4\hat{j})$$
$$= \cancel{6\hat{i}} + 9\hat{j} - \cancel{6\hat{i}} + 8\hat{j}$$
$$\vec{D} = 17\hat{j} = 0\hat{i} + 17\hat{j}$$

Magnitude
$$|\vec{D}| = \sqrt{(0)^2 + (17)^2} = 17$$
$$|\vec{D}| = 17$$

Direction
$$\hat{D} = \vec{D}/|\vec{D}| = \frac{17\hat{j}}{17} = \hat{j}$$
$$\hat{D} = \hat{j}$$

2.7:- Find the angle between the two vectors $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$.

Solution:-
$\vec{A} = 5\hat{i} + \hat{j}$
$\vec{B} = 2\hat{i} + 4\hat{j}$
$\theta = ?$



As we know that

$$\vec{A} \cdot \vec{B} = AB\cos\theta \quad \text{--- (1)}$$

and

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z \quad \text{--- (2)}$$

By comparing eq (1) and (2)

$$AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB} \longrightarrow (3)$$

As $\vec{A} = 5\hat{i} + \hat{j} \Rightarrow A_x = 5,\ A_y = 1,\ A_z = 0$

and $|\vec{A}| = \sqrt{(5)^2 + (1)^2} = \sqrt{26} = 5.09$

Also

$\vec{B} = 2\hat{i} + 4\hat{j} \Rightarrow B_x = 2,\ B_y = 4,\ B_z = 0$

and $|\vec{B}| = \sqrt{(2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20} = 4.47$

Putting above values in eq (3) we get

$$\cos\theta = \frac{(5)(2) + (1)(4) + 0}{(5.09)(4.47)} = \frac{10+4}{(5.09)(4.47)}$$

$$\cos\theta = \frac{14}{22.75}$$

$$\theta = \cos^{-1}(0.615)$$

$$\boxed{\theta = 52^\circ} \quad \textbf{Answer}$$

P. 2.8:- Find the work done when the point of application of the force $3\hat{i} + 2\hat{j}$ moves in a straight line from the pt. (2, -1) to the pt. (6, 4.

Solution:-
$\vec{F} = 3\hat{i} + 2\hat{j}$

Point A (2, -1) $\Rightarrow \vec{r}_A = 2\hat{i} - \hat{j}$

Point B (6, 4) $\Rightarrow \vec{r}_B = 6\hat{i} + 4\hat{j}$

then vector $\vec{r}$ between these to points will be

$$\vec{r} = \vec{r_B} - \vec{r_A} = (6\hat{i} + 4\hat{j}) - (2\hat{i} - \hat{j})$$
$$= 6\hat{i} + 4\hat{j} - 2\hat{i} + \hat{j}$$
$$\vec{r} = 4\hat{i} + 5\hat{j}$$

Thus Work done $= \vec{F} \cdot \vec{r}$

$$= (3\hat{i} + 2\hat{j}) \cdot (4\hat{i} + 5\hat{j})$$
$$= 3\hat{i} \cdot 4\hat{i} + 3\hat{i} \cdot 5\hat{j} + 2\hat{j} \cdot 4\hat{i} + 2\hat{j} \cdot 5\hat{j}$$
$$= 12(\hat{i} \cdot \hat{i}) + 15(\hat{i} \cdot \hat{j}) + 8(\hat{j} \cdot \hat{i}) + 10(\hat{j} \cdot \hat{j})$$
$$= 12 + 10$$
$$\boxed{\text{Work} = 22 \text{ units}}$$ Answer

P. 2.9:- Show that the three vectors $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - 3\hat{j} + 3\hat{k}$ and $4\hat{i} + \hat{j} - 5\hat{k}$ are mutually perpendicular.

Solution:- Let these three vectors are

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$
$$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$$
$$\vec{C} = 4\hat{i} + \hat{j} - 5\hat{k}$$

When these three vectors are perpendicular then

$$\vec{A} \times \vec{B} = \vec{C}$$

This equation means, all the three vectors will be mutually perpendicular when the cross product of $\vec{A}$ and $\vec{B}$ is equal to $\vec{C}$.

$$\therefore \quad \vec{A} \times \vec{B} = (\hat{i} + \hat{j} + \hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k})$$
$$= \hat{i} \times (2\hat{i} - 3\hat{j} + \hat{k}) + \hat{j} \times (2\hat{i} - 3\hat{j} + \hat{k}) + \hat{k} \times (2\hat{i} - 3\hat{j} + \hat{k})$$
$$= 2(\hat{i} \times \hat{i}) - 3(\hat{i} \times \hat{j}) + (\hat{i} \times \hat{k}) + 2(\hat{j} \times \hat{i}) - 3(\hat{j} \times \hat{j}) + (\hat{j} \times \hat{k}) + 2(\hat{k} \times \hat{i}) - 3(\hat{k} \times \hat{j}) + (\hat{k} \times \hat{k})$$

As we know that

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \text{zero}$$

$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$ | $\hat{k} \times \hat{i} = \hat{j}$, $\hat{i} \times \hat{k} = -\hat{j}$

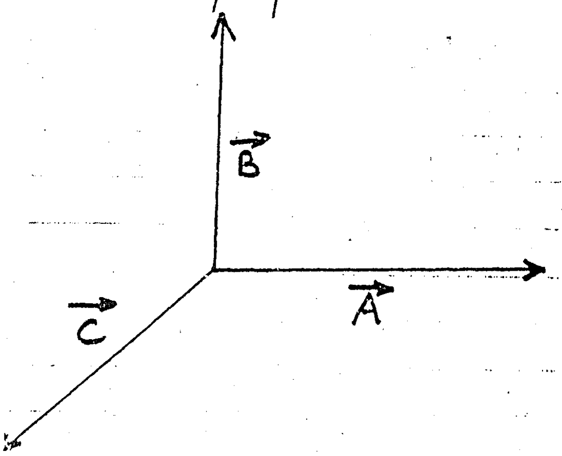
$\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{j} = -\hat{i}$

So $\vec{A}\times\vec{B} = 2(0) - 3\hat{k} - \hat{j} - 2\hat{k} - 3(0) + \hat{i} + 2\hat{j} + 3\hat{i} + 0$

$= 4\hat{i} + \hat{j} - 5\hat{k}$

$\vec{A}\times\vec{B} = \vec{C}$

Hence these three vectors are perpendicular to each other



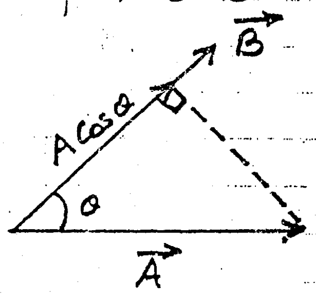
P- 2.10 :-

Given that $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 4\hat{k}$, find the length of the projection of $\vec{A}$ on $\vec{B}$

Solution :-

$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$

and $\vec{B} = 3\hat{i} - 4\hat{k}$.



Projection of $\vec{A}$ on $\vec{B} = A\cos\theta = ?$

We have

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$= B(A\cos\theta)$$

or $$A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{B} = \frac{A_xB_x + A_yB_y + A_zB_z}{B} \quad \text{---} (1)$$

$\vec{B} = 3\hat{i} - 4\hat{k} \Rightarrow B_x = 3,\ B_y = 0,\ B_z = -4$

$|\vec{B}| = \sqrt{(3)^2 + (-4)^2} = \sqrt{25} = 5$

Also $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k} \Rightarrow A_x = 1,\ A_y = -2,\ A_z = 3$

Putting all the values in eq (1), we get

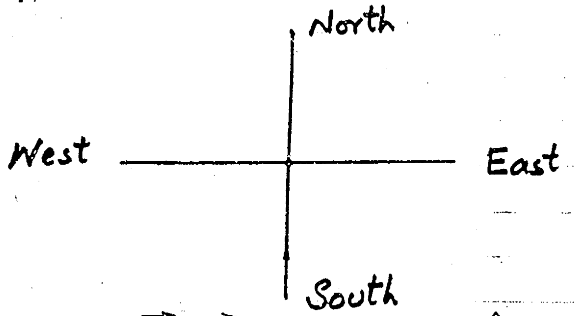
$$A\cos\theta = \frac{(1)(3) + (-2)(0) + (3)(-4)}{5} = \frac{3+0-12}{5}$$

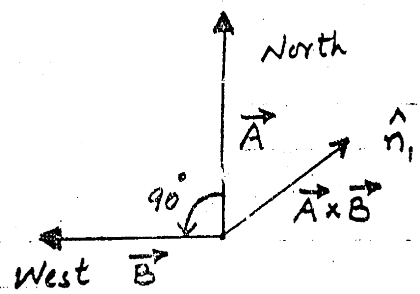
$$\boxed{A\cos\theta = \frac{-9}{5} = \text{Projection of } \vec{A} \text{ on } \vec{B}}$$ **Answer.**

P- 2.11 :- Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 4 units north, 3 units west and 8 units east respectively.
Describe carefully

(a)- $\vec{A}\times\vec{B}$ (b) $\vec{A}\times\vec{C}$ (c) $\vec{B}\times\vec{C}$.

Solution :- $\vec{A}$ = 4 units north
$\vec{B}$ = 3 units west
$\vec{C}$ = 8 units east.





(a) - $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}_1$
$= (4)(3)\sin 90^\circ\,\hat{n}_1$
$\vec{A}\times\vec{B} = 12$ units $\hat{n}_1$

According to right hand rule, curl the finger through the shorter angle i-e from $\vec{A}$ to $\vec{B}$, then thumb will be in vertically up ward.

$\therefore$ $\hat{n}_1$ = vertically up

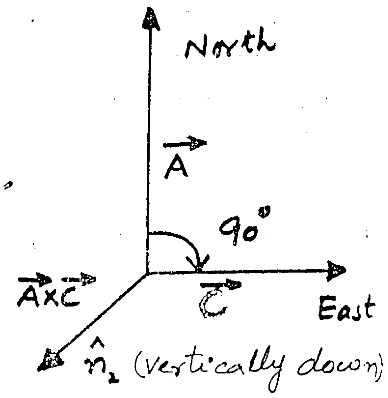
So $\boxed{\vec{A}\times\vec{B} = 12 \text{ units vertically up}}$

(b)-

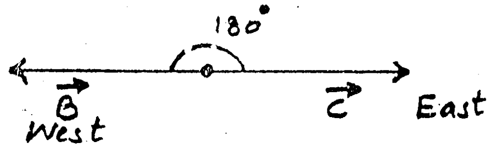
$\vec{A}\times\vec{C} = AC\sin\theta\,\hat{n}_2$
$= (4)(8)\sin 90^\circ\,\hat{n}_2$
$\vec{A}\times\vec{C} = 32$ units $\hat{n}_2$

According to right hand rule,
$\hat{n}_2$ = vertically down

$\therefore$ $\boxed{\vec{A}\times\vec{C} = 32 \text{ units vertically down}}$



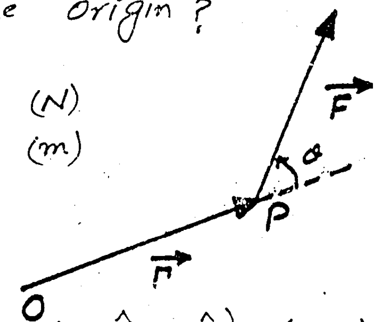
(c)- $\vec{B}\times\vec{C} = BC\sin\theta\,\hat{n}_3$
$\vec{B}\times\vec{C} = (3)(8)\sin 180^\circ\,\hat{n}_3$
$= (3)(8)(0)\,\hat{n}_3$
$\boxed{\vec{B}\times\vec{C} = 0}$



P. 2.12:- The torque or turning effect of force about a given point is given by $\vec{r} \times \vec{F}$ where $\vec{r}$ is the vector from the given point to the point of application of $\vec{F}$. Consider a force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ (newton) acting on the point $7\hat{i} + 3\hat{j} + \hat{k}$ (m). What is the torque in Nm about the origin?

Solution:-

$$\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k} \quad (N)$$
$$\vec{r} = 7\hat{i} + 3\hat{j} + \hat{k} \quad (m)$$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= (7\hat{i} + 3\hat{j} + \hat{k}) \times (-3\hat{i} + \hat{j} + 5\hat{k}) \quad (Nm)$$
$$= 7\hat{i} \times (-3\hat{i} + \hat{j} + 5\hat{k}) + 3\hat{j} \times (-3\hat{i} + \hat{j} + 5\hat{k}) + \hat{k} \times (-3\hat{i} + \hat{j} + 5\hat{k}).$$
$$\vec{\tau} = -21(\hat{i}\times\hat{i}) + 7(\hat{i}\times\hat{j}) + 35(\hat{i}\times\hat{k}) - 9(\hat{j}\times\hat{i}) + 3(\hat{j}\times\hat{j}) + 15(\hat{j}\times\hat{k}) - 3(\hat{k}\times\hat{i}) + (\hat{k}\times\hat{j}) + 5(\hat{k}\times\hat{k}).$$

As we know that

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \text{Zero}$$

and
$$\hat{i}\times\hat{j} = \hat{k}, \quad \hat{j}\times\hat{i} = -\hat{k}$$
$$\hat{j}\times\hat{k} = \hat{i} \quad \hat{k}\times\hat{j} = -\hat{i}$$
$$\hat{k}\times\hat{i} = \hat{j} \quad \hat{i}\times\hat{k} = -\hat{j}$$

Putting these values in above equation we get

$$\vec{\tau} = -21(0) + 7(\hat{k}) + 35(-\hat{j}) - 9(-\hat{k}) + 3(0) + 15(\hat{i}) - 3(\hat{j}) + (-\hat{i}) + 5(0).$$

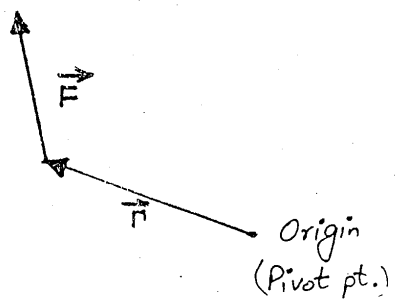
$$\boxed{\vec{\tau} = 14\hat{i} - 38\hat{j} + 16\hat{k} \quad (Nm)}$$ **Answer**

P. 2.13:- The line of action of force $\vec{F} = \hat{i} - 2\hat{j}$ passes through the point whose position vector is $(-\hat{j} + \hat{k})$. Find

(a)- the moment of $\vec{F}$ about the origin.

(b)- the moment of $\vec{F}$ about the point of which the position vector is $(\hat{i} + \hat{k})$.

SOLUTION:-



$\vec{F} = \hat{i} - 2\hat{j}$

$\vec{r} = -\hat{j} + \hat{k}$

(a) - Moment of force about origin $= \vec{\tau} = ?$

$\vec{\tau} = \vec{r} \times \vec{F}$

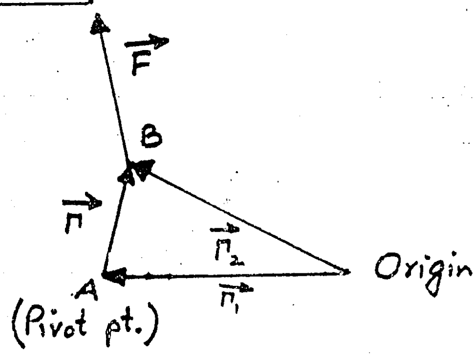
$= (-\hat{j} + \hat{k}) \times (\hat{i} - 2\hat{j})$

$= -(\hat{j} \times \hat{i}) + 2(\hat{j} \times \hat{j}) + (\hat{k} \times \hat{i}) - 2(\hat{k} \times \hat{j})$

$= -(-\hat{k}) + 2(0) + \hat{j} - 2(-\hat{i})$

$$\boxed{\vec{\tau} = 2\hat{i} + \hat{j} + \hat{k}}$$ Answer

(b) In this case



$\vec{r_1} = \hat{i} + \hat{k}$

$\vec{r_2} = -\hat{j} + \hat{k}$

Pivot pt. is 'A' so we have to find out the position vector of force point w.r.to pivot pt. 'A'.

From figure

$\vec{r_1} + \vec{r} = \vec{r_2}$ By head to tail rule

$\therefore \vec{r} = \vec{r_2} - \vec{r_1}$

$= (-\hat{j} + \hat{k}) - (\hat{i} + \hat{k})$

$= -\hat{j} + \cancel{\hat{k}} - \hat{i} - \cancel{\hat{k}}$

$\vec{r} = -\hat{i} - \hat{j}$

$\therefore$ The moment of force about point 'A' is

$\vec{\tau}' = \vec{r} \times \vec{F}$

$= (-\hat{i} - \hat{j}) \times (\hat{i} - 2\hat{j})$

$= -(\hat{i} \times \hat{i}) + 2(\hat{i} \times \hat{j}) - (\hat{j} \times \hat{i}) + 2(\hat{j} \times \hat{j})$

$= -0 + 2\hat{k} - (-\hat{k}) + 2(0)$

$$\boxed{\vec{\tau}' = 3\hat{k}}$$

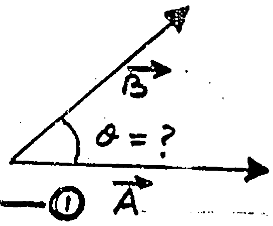
P. 2.14 :- The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors.

SOLUTION :- Let $\vec{A}$ and $\vec{B}$ be the two vectors.

Dot Product $= \vec{A} \cdot \vec{B} = 6\sqrt{3}$

Cross Product $= \vec{A} \times \vec{B} = 6$

The angle between them $= \theta = ?$



As we have

$$\vec{A} \cdot \vec{B} = AB\cos\theta \quad \text{——— (1)}$$

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n} \quad \text{——— (2)}$$

Squaring equation (1) we get

$$(\vec{A} \cdot \vec{B})^2 = (AB\cos\theta)^2$$

$$(\vec{A} \cdot \vec{B})^2 = A^2B^2\cos^2\theta \quad \text{——— (3)}$$

Now squaring eq. (2) we get

$$(|\vec{A}\times\vec{B}|)^2 = [\vec{A}\times\vec{B}]^2 = (\vec{A}\times\vec{B}) \cdot (\vec{A}\times\vec{B})$$

$$(\vec{A}\times\vec{B})^2 = (AB\sin\theta\,\hat{n}) \cdot (AB\sin\theta\,\hat{n}) \quad \because$$

$$= A^2B^2\sin^2\theta\,(\hat{n}) \cdot (\hat{n}) \quad (\because V^2 = \vec{V} \cdot \vec{V})$$

$$= A^2B^2\sin^2\theta\,(\hat{n} \cdot \hat{n})$$

$$= A^2B^2\sin^2\theta\,(1.1.\cos 0^\circ)$$

$$(\vec{A}\times\vec{B})^2 = A^2B^2\sin^2\theta \quad \text{——— (4)}$$

Adding eq (3) and (4) we get

$$(\vec{A} \cdot \vec{B})^2 + (\vec{A}\times\vec{B})^2 = A^2B^2\cos^2\theta + A^2B^2\sin^2\theta$$

$$= A^2B^2(\cos^2\theta + \sin^2\theta)$$

$$= A^2B^2$$

Taking square root on both the sides.

$$AB = \sqrt{(\vec{A} \cdot \vec{B})^2 + (\vec{A}\times\vec{B})^2}$$

$$= \sqrt{(6\sqrt{3})^2 + (6)^2}$$

$$= \sqrt{(36\times 3) + 36} = \sqrt{108 + 36}$$

$$AB = \sqrt{144} = 12$$

(73)

Now $\vec{A}\cdot\vec{B} = AB\cos\theta$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\theta = 30^\circ}$$ **Answer.**

Q 2.15: A load of 10N is suspended from a clothes line. This distorts the line so that it makes an angle of 15° with the horizontal at each end. Find the tension in the clothes line.

SOLUTION:

Load $= W = 10N$

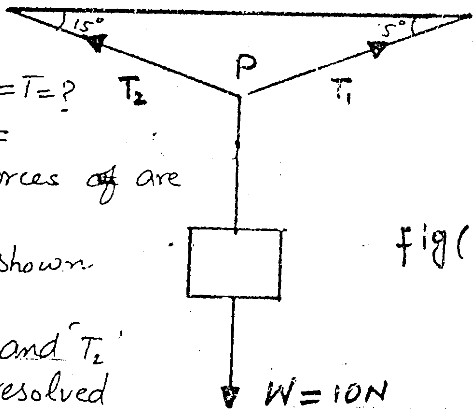
Angle $= \theta = 15^\circ$

Tension in the clothesline $= T = ?$

For using condition of equilibrium, all the forces are acting on point 'P'.

The force diagram is shown in figure (a).

The inclined forces 'T_1' and 'T_2' can now be easily resolved along x and y-axis as shown in fig (b)



fig(a)

Applying

$\Sigma F_x = 0$

$T_1\cos15^\circ - T_2\cos15^\circ = 0$

$T_1\cos15^\circ = T_2\cos15^\circ$

$T_1 = T_2 = T$ (let)

Now applying

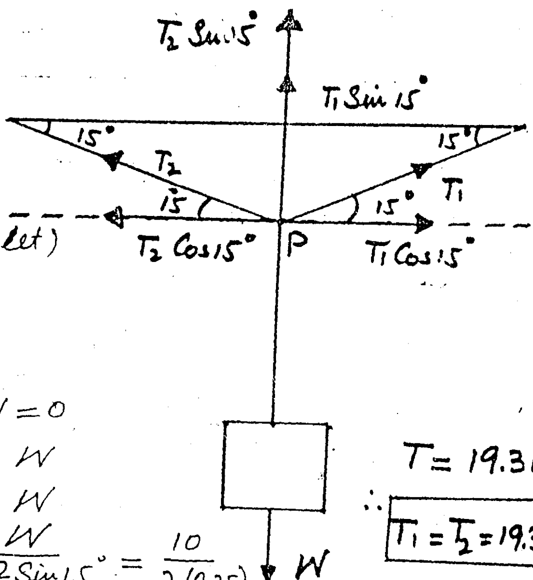
$\Sigma F_y = 0$

$T_1\sin15^\circ + T_2\sin15^\circ - W = 0$

$T\sin15^\circ + T\sin15^\circ = W$

$2T\sin15^\circ = W$

$T = \dfrac{W}{2\sin15^\circ} = \dfrac{10}{2(0.25)}$

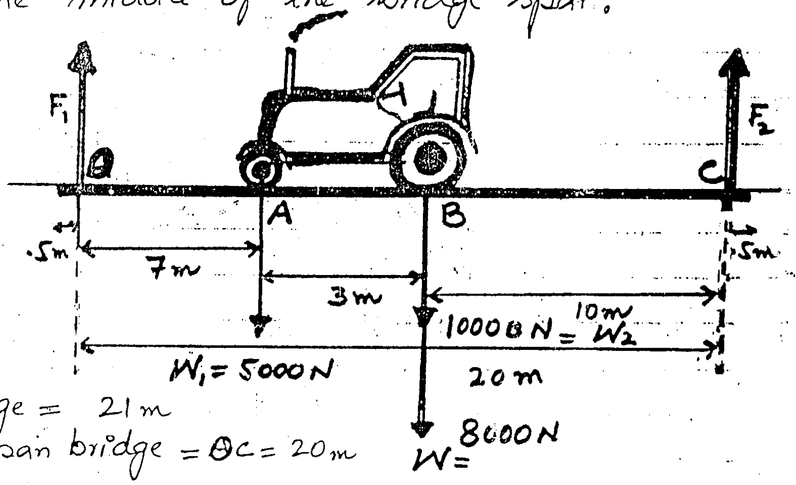


$T = 19.3N$

$\therefore \boxed{T_1 = T_2 = 19.3N}$

2.16 :. A tractor of weight 15,000N crosses a single span bridge of weight 8000 N and of length 21.0 m. The bridge span is supported half a metre from either end. tractor's front wheels take 1/3 of the total weight of the tractor, and the rear wheels are 3m behind the front wheels. Calculate the force on the bridge supports when the rear wheels are at the middle of the bridge span.

SOLUTION :-



Weight of tractor
$W' = 15{,}000N$
Weight of bridge
$W = 8000N$

Length of bridge = 21 m
Length of the span bridge = $OC = 20m$

Weight of front wheel of the tractor $W_1 = \frac{1}{3} \times W'$

$= \frac{1}{3} \times 15000$

$= 5000\ N$

Weight of rear wheel of the tractor $W_2 = W' - W_1$

$= 15000 - 5000$

$= 10{,}000N$

Distance between two wheels $= AB = 3m$

Forces on the bridge supports $= F_1 = ?$

$F_2 = ?$

When the rear wheel of the tractor is in middle of the span bridge so $BC = 10m$

According to figure $AB = 3m$

and $OA = 7m$

Now applying first condition of equilibrium

$\Sigma F_x = 0$

and $\Sigma F_y = 0$

As there is no force acting along x-axis so apply

$$\Sigma F_y = 0$$

$$F_1 + F_2 - W_1 - W_2 - W = 0$$

$$F_1 + F_2 - 5000 - 10,000 - 8000 = 0$$

$$F_1 + F_2 = 23,000 N \quad \text{---} \quad (1)$$

Now applying second condition of equilibrium

$$\Sigma \tau = 0$$

Take 'O' as pivot point.

$\therefore$ Moment arm of $F_1 = 0$

Moment arm of $F_2 = OC$

Using $\tau = (\text{Force})(\text{Moment arm})$

$$F_1(0) + F_2(OC) - W_1(OA) - W_2(OB) - W(OB) = 0$$

$$F_1(0) + F_2(20) - 5000(7) - 10000(10) - 8000(10) = 0$$

$$20 F_2 - 35000 - 100000 - 80000 = 0$$

$$20 F_2 = 215000 N$$

$$F_2 = \frac{215000}{20}$$

$$F_2 = 10750 N = 10.750 \times 10^3 N$$

$$\boxed{F_2 = 10.75 KN} \quad \text{Answer}$$

Putting this value in eq (1)

$$F_1 = 23000 - F_2$$

$$= 23000 - 10750$$

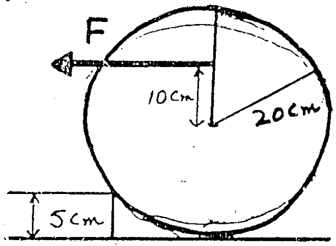
$$F_1 = 12250 N = 12.250 \times 10^3 N$$

$$\boxed{F_1 = 12.25 KN} \quad \text{Answer}$$

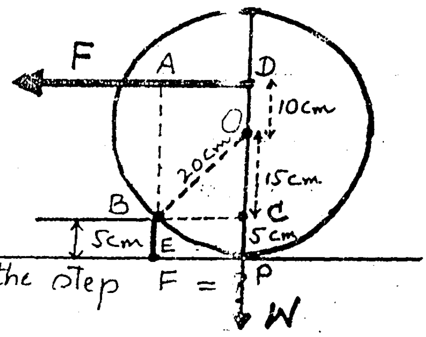
P 2.17 :- A spherical ball of weight 50N is to be lifted over the step as shown in fig.

(a) - Calculate the minimum force needed just to lift it above the floor.

(b) Determine the force acting on the ball at that instant.



SOLUTION :-



Weight of the spherical ball

$W = 50N$

The curl height $h = 5cm$

Radius of ball $= r = 20cm$

(a) Minimum force needed to lift over the step $F = ?$

As from figure

$BE = h = 5cm = CP$

Radius $OB = r = OP = 20cm$

And $OC = 15cm$ $(\because OC = OP - PC)$

$OD = 10cm$

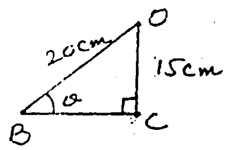
and $DC = OC + OD = 15 + 10$

$= 25cm$

As from figure $DC = AB = 25cm$

From rt $\angle$d $\triangle$ OBC

By Pythagorean's theorm



$(OP)^2 = (BC)^2 + (OC)^2$

$(BC)^2 = (OB)^2 - (OC)^2$

$(BC)^2 = (20)^2 - (15)^2$

$BC = \sqrt{400 - 225} = \sqrt{175} = 13.2cm$.

Now taking 'B' as pivot pt.

$\Sigma \tau_B = 0$

Using Torque = (Force)(Moment Arm)

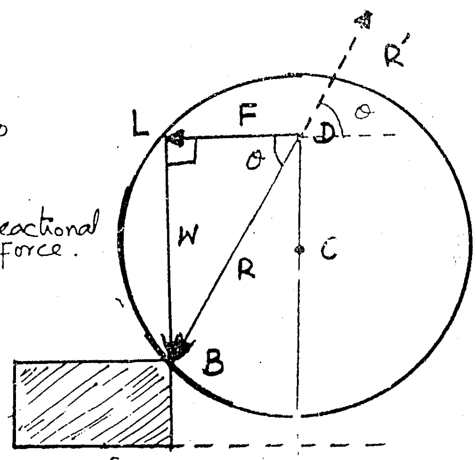
$F(AB) - W(BC) = 0$

$F(25) - 50(13.2) = 0$

$F = \dfrac{50 \times 13.2}{25}$

$= 26.4 N$

$\boxed{F \cong 26N}$ Answer.



(b)- Our requirement is to calculate the resultant force 'R'. Let R' be its reactional force. In this case weight of the ball will act at point 'B'. Two forces are acting on this ball weight and applied force 'F'.

By head-to-tail rule it can be seen that $\vec{R}$ is the resultant force. i-e

$$\vec{R} = \vec{F} + \vec{W}.$$

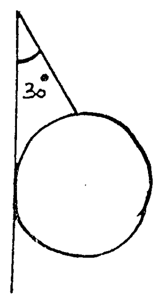
From figure we have

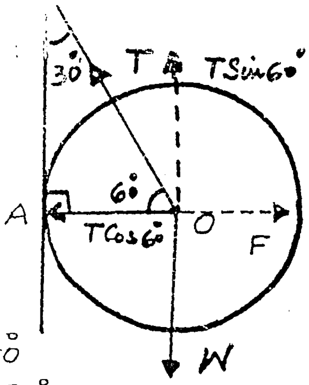
$$LD = F = 26\,N$$
$$LB = W = 50\,N$$
$$BD = R = ?$$

So applying Pythagorean theorm on rt∠d △DLB

$$(BD)^2 = (LD)^2 + (LB)^2$$
$$R^2 = (F)^2 + (W)^2$$
$$R = \sqrt{F^2 + W^2} = \sqrt{(26)^2 + (50)^2}$$
$$R = 56.3\,N$$
$$\boxed{R \simeq 56\,N}$$

P. 2.18 :- A uniform sphere of weight 10N is held by a string attached to a frictionless wall so that the string makes an angle of 30° with the wall as shown in fig. Find the tension in the string and the force exerted on the sphere by the wall.





Solution:

Weight of sphere $= W = 10\,N$

$\angle O\hat{B}A = 30^\circ$

Tension in the string $= T = ?$

Force exerted by wall on sphere $= F = ?$

Now consider rt ∠d $\triangle OAB$

$$\angle O\hat{A}B + \angle O\hat{B}A + \angle A\hat{O}B = 180^\circ$$

$$90^\circ + 30^\circ + \angle A\hat{O}B = 180^\circ$$

$$\therefore \quad \angle A\hat{O}B = \theta = 180^\circ - 120^\circ = 60^\circ$$

Now resolving Tension into its rectangular components:

There are four forces are acting.

Two are $T\cos 60^\circ$ and its reactional force F along x-axis, while other two '$T\sin 60^\circ$' and weight along y-axis.

Now applying first condition of equilibrium

$$\Sigma F_x = 0$$

$$F - T\cos 60^\circ = 0$$

$$F = T\cos 60^\circ \quad \text{——— } (1)$$

Also

$$\Sigma F_y = 0$$

$$T\sin 60^\circ - W = 0$$

$$T\sin 60^\circ = W$$

$$T = \frac{W}{\sin 60^\circ} = \frac{10\,N}{\sin 60^\circ}$$

$$\boxed{T = 11.5\,N} \quad \text{Answer}$$

Putting this value in eq (1) we get

$$F = (11.5\,N)\cos 60^\circ$$

$$\therefore \quad \boxed{F = 5.77\,N} \quad \text{Answer}$$

This is the force exerted by wall on the sphere.