PROBLEMS CH# 3 (40)

PROBLEM 3.1

A helicopter is ascending vertically at the rate of $19.6\ ms^{-1}$. when it is at a height of 156.8m above the ground, a stone is dropped. How long does the stone take to reach the ground.

-: SOLUTION :- If the direction of initial velocity is taken as positive then the direction of other quantities like distance and acceleration due to gravity will be negative as they are opposing the direction of V_i

There fore

$$V_i = 19.6\ m\,sec^{-1}$$
$$S = -156.8\ m$$
$$g = -9.8\ m\,sec^{-2}$$
$$t = ?$$

By using 2nd equation of motion

$$S = v_i t + \frac{1}{2} g t^2$$
$$-156.8 = 19.6t + \frac{1}{2}(-9.8)t^2$$
$$-156.8 = 19.6t - 4.9t^2$$

÷ing by 4.9 on both sides

$$-32 = 4t - t^2$$
$$\Rightarrow \quad t^2 - 4t - 32 = 0$$
$$t^2 - 8t + 4t - 32 = 0$$
$$t(t-8) + 4(t-8) = 0$$
$$(t+4)(t-8) = 0$$

$t+4=0 \Rightarrow t=-4\text{ sec}$

or $t-8=0 \Rightarrow t=8\text{ sec}$

As time is always +ve so the acceptable solution is $t = 8$ sec Ans.

PROBLEM # 3.2 :-

Using the following data, draw a velocity time graph for a short journey on a straight road of a motorbike

Velocity (ms^{-1})	0	10	20	20	20	20	0
Time (s)	0	30	60	90	120	150	180

Use the graph to calculate :-

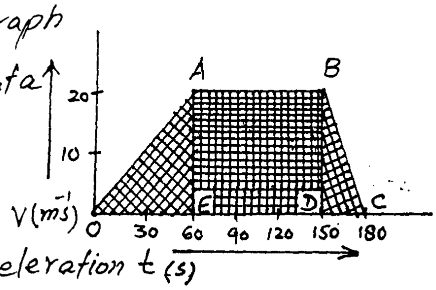
(a) The initial acceleration

(b) The final acceleration

(c) The total distance traveled by the motorcyclist.

-: SOLUTION :-

The velocity time graph according to the given data is shown in the figure.



(a) The initial acceleration is equal to the slope of line OA

$$\therefore \quad a = \frac{AE}{OE} = \frac{\Delta V}{t} = \frac{20}{60} = 0.33\, ms^{-2}$$

(b) The final acceleration is equal to the slope of line (graph) BC

$$\therefore \quad a' = \frac{BD}{DC} = \frac{\Delta V}{t} = \frac{20}{30} = 0.66\, ms^{-2}$$

(c) The total distance covered by the motercyclist is equal to the area of shaded region under velocity time graph with time axis.

So

$$S = \text{Area of Shaded region}$$
$$= \text{Area of } \Delta AOE + \text{Area of rectangle ABDE} + \text{Area of } \Delta BCD$$
$$= \frac{1}{2} \times V \times t + V \times t + \frac{1}{2} \times V \times t$$
$$= \frac{1}{2} \times 20 \times 60 + 20 \times 90 + \frac{1}{2} \times 20 \times 30$$
$$= 600 + 1800 + 300 = 2700m = 2.7\ Km$$

ALTERNATE METHOD

As the figure is a trapezium. So its area is given as

$$S = \text{Area} = \frac{1}{2} \times (\text{Sum of parallel sides}) \times (\text{Separation of parallel sides})$$
$$= \frac{1}{2} \times (180 + 90) \times 20$$
$$= 270 \times 10 = 2700m = 2.7\ Km$$

PROBLEM 3.3

A proton moving with speed of 1.0×10^{7} m/sec passes through a 0.02 cm thick sheet of paper and emerges with a speed of 2.0×10^{6} m/sec. Assuming uniform deceleration, Find retardation and time taken to pass through the paper.

-: SOLUTION :-

-: Data:-

$V_i = 1.0 \times 10^7 ms^{-1}$

$V_f = 2.0 \times 10^6 ms^{-1}$

$S = 0.02 cm = 0.02 \times 10^{-2} m = 2 \times 10^{-4} m$

$a = ?$

$t = ?$

For a using 3rd equation of motion

$$2aS = v_f^2 - v_i^2$$

$$2 \times a \times 2 \times 10^{-4} = (2 \times 10^6)^2 - (1 \times 10^7)^2$$

$$4 \times 10^{-4} a = 4 \times 10^{12} - 1 \times 10^{14}$$

$$a = \frac{4 \times 10^{12} - 100 \times 10^{12}}{4 \times 10^{-4}}$$

$$= \frac{10^{12}(4 - 100)}{4 \times 10^{-4}} = \frac{-96 \times 10^{12}}{4 \times 10^{-4}}$$

$$= -24 \times 10^{16} ms^{-2}$$

or $a = -2.4 \times 10^{17} ms^{-2}$

where -ve sign shows retardation

For t using Ist equation of motion

$$v_f = v_i + at$$

$$2 \times 10^6 = 1 \times 10^7 + (-2.4 \times 10^{17}) t$$

$$+2.4 \times 10^{17} t = 1 \times 10^7 - 2 \times 10^6$$

$$t = \frac{10 \times 10^6 - 2 \times 10^6}{2.4 \times 10^{17}}$$

$$= \frac{10^6 (10 - 2)}{2.4 \times 10^{17}} = \frac{8 \times 10^6}{2.4 \times 10^{17}}$$

$$= 3.33 \times 10^{-11} sec.$$

-: PROBLEM # 3.4 :-

Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What is the ratio of their velocities after the spring has been released?

-: SOLUTION :-

$$\text{ratio of velocities} = \frac{v_1}{v_2} = ?$$

As the masses m_1 and m_2 are at rest at the ends of a compressed spring. So

$$\text{Total initial momentum} = P_i = 0$$

When the spring is released both the masses m_1 and m_2 tend to move with velocities v_1 and v_2 respectively in opposite direction. Their final momentum is

$$P_f = m_1v_1 + m_2v_2$$

According to the law of Conservation of momentum

$$P_i = P_f$$

$$0 = m_1v_1 + m_2v_2$$

$$\Rightarrow m_1v_1 = -m_2v_2$$

$$\text{or} \quad \frac{v_1}{v_2} = -\frac{m_2}{m_1}$$

-: PROBLEM # 3.5 :-

An amoeba of mass 1.0×10^{-12} kg propels itself throw water by blowing a jet of water through a tiny orifice. The amoeba ejects water with a speed of $1.0\times10^{-4}\ ms^{-1}$ and at a rate of $1.0 \times 10^{-13}\ kg\,s^{-1}$. Assume that the water is being continuously replenished so that the mass of amoeba remains the same.

(a) If amoeba moves with constant velocity through water. what is the force of surrounding water on amoeba?

(b) If there were no force on amoeba other than the reactional force caused by the emerging jet, what would be the acceleration of amoeba?

-: SOLUTION :- Data

mass of amoeba $= m = 1.0 \times 10^{-12}\ kg$

Speed of ejecting water $= v = 1.0 \times 10^{-4}\ m\,s^{-1}$

mass per second of water $= 1.0 \times 10^{-13}\ kg\,s^{-1}$

(a) $F = ?$

(b) $a = ?$

(a) The force exerted on amoeba is equal and opposite to the force due the blowing of jet of water which is given as

$$F = \text{mass per second} \times \text{Speed of water ejected}$$
$$= 1.0 \times 10^{-13}\ kg\,s^{-1} \times 1.0 \times 10^{-4}\ m\,s^{-1}$$
$$F = 1.0 \times 10^{-17}\ N$$

(b) If the above force produces an acceleration a in amoeba of mass m then

$$F = ma$$
$$\Rightarrow a = \frac{F}{m} = \frac{1.0 \times 10^{-17}}{1.0 \times 10^{-12}}$$
$$a = 1.0 \times 10^{-5}\ m\,s^{-2}$$

PROBLEM # 3.6 :-

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A boy places cracker of negligible mass in an empty can of 40 g mass. He plugs the end with a wooden block of mass 200g. After igniting the fire cracker, he throws the can straight up. it explodes at the top of its path. If the block shoots out with a speed of $3ms^{-1}$. How fast will a can be going.

SOLUTION :-

Mass of can $= m_1 = 40g = 40\times10^{-3}kg$

Mass of block $= m_2 = 200g = 0.2kg$

Final velocity of block $= V_2 = 3ms^{-1}$

Final velocity of can $= V_1 = ?$

As block and the can are at rest. therefore their initial momentum is equal to zero i.e $P_i = 0$

After explosion the final momentum of can and block is given as

$$P_f = m_1V_1 + m_2V_2$$

According to the law of conservation of momentum

$$P_f = P_i$$

$$m_1V_1 + m_2V_2 = 0$$

$$40\times10^{-3}\times V_1 + 0.2\times3 = 0$$

$$40\times10^{-3}V_1 = -0.2\times3$$

$$V_1 = \frac{-0.6}{40\times10^{-3}}$$

$$= -\frac{6\times10^3}{400} = \frac{-\cancel{6000}^{15}}{\cancel{400}}$$

$$V_1 = -15ms^{-1}$$

Ans.

PROBLEM # 3.7 (47)

An electron ($m = 9.1 \times 10^{-31}$ Kg) travelling at $2.0 \times 10^{7}\ ms^{-1}$ undergoes a head on collision with a hydrogen atom ($m = 1.67 \times 10^{-27}$ Kg) which is initially at rest. Assuming the collision to be perfectly elastic and a motion to be along a same straight line. Find the velocity of hydrogen atom?

-: SOLUTION :-

mass of electron $= m_1 = 9.1 \times 10^{-31}$ Kg

mass of proton $= m_2 = 1.67 \times 10^{-27}$ Kg

velocity of electron $= V_1 = 2.0 \times 10^{7}\ ms^{-1}$

velocity of proton $= V_2 = 0$

velocity of proton after collision $= V_2' = ?$

In an elastic collision the velocity of target body after collision is given as

$$V_2' = \frac{2m_1 V_1}{m_1 + m_2} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) V_2$$

$$= \frac{2 \times 9.1 \times 10^{-31}}{9.1 \times 10^{-31} + 1.67 \times 10^{-27}} \times 2.0 \times 10^{7} \qquad (\because V_2 = 0)$$

$$= 2.18 \times 10^{4}\ ms^{-1}$$

PROBLEM # 3.8

A truck weighing 2500 Kg and moving with a velocity of $21\ ms^{-1}$ collides with a stationary car weighing 1000 Kg. The truck and the car move together after the impact. Calculate their common velocity.

SOLUTION -: DATA :-

mass of the truck $= m_1 = 2500$ Kg

Initial Velocity of the truck = $V_1 = 21 m s^{-1}$

Mass of the Car = $m_2 = 1000 kg$

Initial velocity of the Car = $V_2 = 0$

Common velocity of car and truck = $V = ?$

By using the law of Conservation of momentum

Initial momentum = Final momentum

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$$

$$\because V_1' = V_2' = V$$

$$\therefore m_1V_1 + m_2V_2 = m_1V + m_2V$$

$$2500 \times 21 + 1000 \times 0 = 2500V + 1000V$$

$$52500 = 3500V$$

$$V = \frac{525\cancel{00}}{35\cancel{00}}$$

$$V = 15 ms^{-1}$$

PROBLEM # 3.9

Two blocks of masses 2.0kg and 0.5kg are attached at the two ends of a Compressed Spring. The elastic potential energy stored in the spring is 10J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.

-: SOLUTION :-

Mass of Ist block = $m_1 = 0.5kg$

Mass of 2nd block = $m_2 = 2.0 kg$

P.E. stored in the Spring = 10J

Velocity of Ist block = $V_1 = ?$

Velocity of 2nd block = $V_2 = ?$

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When the blocks are moving in opposite direction the energy of the spring is transfered to the block as their kinetic energy. Hence according to the law of Conservation of E

energy gained by blocks = energy lost by spring

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = 10$$

$$m_1 v_1^2 + m_2 v_2^2 = 20$$

$$0.5\, v_1^2 + 2\, v_2^2 = 20$$

$$v_1^2 + 4 v_2^2 = 40 \quad (1) \text{ xing by 2}$$

By using the law of Conservation of momentum.

Initial momentum = Final momentum

$$0 = m_1 v_1 + m_2 v_2$$

$$0 = 0.5\, v_1 + 2\, v_2$$

$$0.5\, v_1 = -2\, v_2$$

$$v_1 = -4 v_2 \quad (2)$$

Using the value of v_1 in equation 1

$$(-4v_2)^2 + 4 v_2^2 = 40$$

$$16 v_2^2 + 4 v_2^2 = 40 \Rightarrow 20 v_2^2 = 40$$

$$v_2^2 = 2 \Rightarrow v_2 = 1.41\, ms^{-1}$$

Hence $v_2 = 1.41\, ms^{-1}$

$$\because v_1 = -4 v_2$$

$$\Rightarrow v_1 = -4 \times 1.41$$

$$v_1 = -5.64\, m s^{-1}$$

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-: PROBLEM # 3.10

A football is thrown upward with an angle of 30° with respect to the horizontal. To throw a 40 m pass, what must be the initial velocity of the ball?

-: SOLUTION :- DATA

$\theta = 30°$, $R = 40m$, $g = 9.8 ms^{-2}$

$v_i = ?$

$$\because \quad R = \frac{v_i^2}{g} \sin 2\theta$$

$$40 = \frac{v_i^2}{9.8} \times \sin 2 \times 30$$

$$v_i^2 = \frac{40 \times 9.8}{\sin 60} = \frac{392}{0.866}$$

$$v_i^2 = 452.65$$

$$\Rightarrow v_i = 21.27 \ ms^{-1}$$

$$v_i \simeq 21.3 \ ms^{-1}$$

PROBLEM # 3.11

A ball is thrown horizontally from a height of 10m with a velocity of $21 ms^{-1}$. How far off it hit the ground and with what velocity?

-: SOLUTION :-—: Data:-

$Y = h = 10m$

$v_i = 21 \ ms^{-1}$.

$\theta = 0°$ (Along x-axis)

$g = 9.8 \ ms^{-2}$

$t = ?$

$X = ?$

$V_f = ?$

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$\because \quad Y = v_{iy}t + \frac{1}{2}gt^2$

$10 = v_i \sin\alpha + \frac{1}{2}gt^2$

$10 = 21 \sin 0 + \frac{1}{2} \times 9.8\, t^2$

$10 = 4.9t^2 \qquad (\because \sin 0 = 0)$

$t^2 = \frac{10}{4.9}$

$\Rightarrow t = \sqrt{\frac{10}{4.9}} = 1.428 \text{ sec}$

As $X = v_{ix} t = v_i \cos\alpha \, t$

$\Rightarrow X = 21 \times \cos 0 \times 1.428$

$= 29.98 m$

$X \simeq 30 m$

For v_f

$v_{fx} = v_{ix} = v_i \cos\alpha = 21 \cos 0 = 21 ms^{-1}$

$v_{fy} = v_{iy} + gt$

$= v_i \sin\alpha + gt$

$= 21 \sin 0 + 9.8 \times 1.428$

$= 0 + 13.99$

$v_{fy} \simeq 14 ms^{-1}$

$\therefore \quad v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$

$= \sqrt{(21)^2 + (14)^2}$

$v_f = 25.2 \, m s^{-1}$

$v_f \simeq 25 m s^{-1}$

PROBLEM # 3.12

A bomber dropped a bomb at a height of 490m when its velocity along the horizontal was 300 kmh^{-1}.

(a) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground?

(b) How long was it in air?

-: SOLUTION :-

$$Y = h = 490\,m$$
$$v_i = 300\,Km\,h^{-1} = 83.3\,m\,s^{-1}$$
$$g = 9.8\,m\,s^{-2}$$
$$t = ?$$
$$x = ?$$

using 2nd equation of motion along vertical direction

$$Y = v_{iy} t + \frac{1}{2} g t^2$$
$$Y = v_i \sin\theta\, t + \frac{1}{2} g t^2$$
$$490 = 83.3 \times \sin 0 \times t + \frac{1}{2} \times 9.8\, t^2$$
$$490 = 4.9 t^2$$
$$\Rightarrow t^2 = \frac{490}{4.9} = 100$$
$$t = 10\,sec$$

For horizontal distance

$$X = v_{ix} t = v_i \cos\theta\, t$$
$$X = 83.3 \times \cos 0 \times 10$$
$$X = 833\,m \quad \text{Ans.}$$

PROBLEM # 3.13 :-

Find the angle of a projectile for which its maximum height and horizontal range are equal.

-: SOLUTION :- $\theta = ?$

According to the question

$$H_{max} = R$$

$$\frac{\cancel{v_i^2}\sin^2\alpha}{2\cancel{g}} = \frac{\cancel{v_i^2}}{\cancel{g}}\sin 2\alpha \qquad (53)$$

$$\frac{\sin^{\cancel{2}}\alpha}{2} = 2\cancel{\sin\alpha}\cos\alpha$$

$$\frac{\sin\alpha}{2} = 2\cos\alpha$$

$$\Rightarrow \quad \frac{\sin\alpha}{\cos\alpha} = 4$$

or $\quad \tan\alpha = 4$

$$\alpha = \tan^{-1}(4)$$

$$\alpha = 75.96^\circ$$

$$\theta \simeq 76^\circ$$

:PROBLEM # 3.14 :–

Prove that for angles of projection, which exceed or fall short of 45° by equal amounts, the ranges are equal.

SOLUTION :–

According to the given Condition the range of projectile should be same for the following angle of projection

$$\theta = (45 \pm \phi)$$

Let $\phi = 15^\circ$ then the required angles of projection are $45+15 = 60^\circ$ & $45-15 = 30^\circ$ so the first range concerning to 60° is

$$R_1 = \frac{v_i^2}{g}\sin 2\theta$$

$$= \frac{v_i^2}{g}\sin 2\times 60$$

$$= \frac{v_i^2}{g}\sin 120 = 0.866\frac{v_i^2}{g}$$

or $\quad R_1 = 0.088 v_i^2 \quad \text{——} (1)$

2nd range concerning to the other angle equal to 30° is

(54)

$$R_2 = \frac{v_i^2}{g}\sin 2\theta = \frac{v_i^2}{g}\sin 2\times 30$$

$$= \frac{v_i^2}{g}\sin 60^\circ = 0.866\frac{v_i^2}{g}$$

or $R_2 = 0.088\, v_i^2$ ——— (2)

From eq (1) and eq (2) the statement is verified.

PROBLEM # 3.15

A SLBM (Submarine launched ballistic missile) is fired from a distance of 3000 Km. If the earth were flat and the angle of launch is 45° with horizontal. Find the time taken by SLBM to hit the target and the velocity with which the missile is fired.

-: SOLUTION:-

DATA:-

$R = 3000\,Km = 3000\times 1000\,m = 3\times 10^6\,m$

$\theta = 45^\circ$, $g = 9.8\,m\,s^{-2}$

$v_i = ?$

$t = ?$

For v_i

$$R = \frac{v_i^2}{g}\sin 2\theta$$

$$\Rightarrow v_i^2 = \frac{Rg}{\sin 2\theta}$$

$$v_i^2 = \frac{3\times 10^6 \times 9.8}{\sin 2\times 45} = \frac{29.4\times 10^6}{\sin 90}$$

$$v_i = \sqrt{29.4\times 10^6} = 5422\,m\,sec^{-1}$$

$$= 5.42\times 10^3\,m\,sec^{-1} = 5.42\,km\,s^{-1}$$

For t

$$t = \frac{2v_i\sin\theta}{g} = \frac{2\times 5.42\times 10^3\times \sin 45}{9.8}$$

$$= 782\,S$$

$t = 13\,min$ Ans.