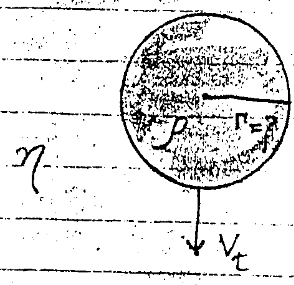
PROBLEMS

P. 6.1 :- Certain globular protein particle has a density of 1246 $Kg\,m^{-3}$. It falls through pure water ($\eta = 8.0\times10^{-4}\,N\,m\,s^{-2}$) with a terminal speed of $3.0\,cm\,h^{-1}$. Find the radius of particle.

Solution :-

Data :-



Density of protein particle =

$\rho = 1246\,Kg\,m^{-3}$

Coefficient of viscosity of water

$\eta = 8.0\times10^{-4}\,N\,m\,s^{-2}$

$= 8.0\times10^{-4}$

$V_t = 3\,cm\,h^{-1} = \dfrac{3\times10^{-2}}{3600}\,m\,s^{-1}$

$V_t = 8.33\times10^{-6}\,m\,s^{-1}$.

Radius $r = ?$

Calculations

As we have

$$V_t = \frac{2 g r^2 \rho}{9\eta}$$

$$r^2 = \frac{9\eta v_t}{2g\rho}$$

$$r^2 = \frac{9\times 8\times 10^{-4}\, N m s^{-2} \times 8.33\times 10^{-6}\, m s^{-1}}{2\times 9.8\, m s^{-2}\times 1246\, Kg\, m^{-3}}$$

$$r = \sqrt{25\times 10^{-10}}\ m$$

$$\boxed{r = 5\times 10^{-5}\ m}$$ Answer.

P. 6.2 :- Water flows through a hose, whose internal diameter is 1cm, at a speed of $1ms^{-1}$. What should be the diameter of the nozzle if the water is to emerge at $21ms^{-1}$?

Solution

Data :- $d_1 = 1cm = 10^{-2} m$

speed of water flow $v_1 = 1ms^{-1}$
Speed of water emergence $V_2 = 21ms^{-1}$
Diameter of the nozzle $= d_2 = ?$

Calculation :-

According to equation of continuity

$$A_1 V_1 = A_2 V_2 \quad \text{———— } (1)$$

where $A = \pi r^2$

So $A_1 = \pi r_1^2 = \pi\left(\frac{d_1}{2}\right)^2 \quad (\because d = 2r)$

$$A_1 = \frac{\pi d_1^2}{4}$$

Similarly $A_2 = \frac{\pi d_2^2}{4}$

Putting in eq (1)

$$\frac{\cancel{\pi} d_1^2}{\cancel{4}} v_1 = \frac{\cancel{\pi} d_2^2}{\cancel{4}} v_2$$

$$d_1^2 v_1 = d_2^2 v_2$$

$$d_2^2 = \frac{d_1^2 v_1}{v_2}$$

$$d_2 = \sqrt{\frac{v_1}{v_2} d_1^2}$$

$$= \sqrt{\frac{1}{21} \times (.01)^2} = \sqrt{.05} \times (.01)$$

$$= 0.002 \text{ meters}$$

$$\boxed{d_2 = 0.2 \text{ cm}}$$ Answer

P. 6.3 :- The pipe near the lower end of a large water storage tank develops a small leak and a stream of water shoots from it. The top of water in the tank is 15 m above the point of leak.

(a). With what speed does the water rush from the hole?

(b). If the hole has an area of 0.060 cm^2, how much water flows out in one second?

SOLUTION :-

Data :- Height of water $= \Delta h = 15 \text{ m}$

Area of hole $= A = 0.06 \text{ cm}^2$

Speed of water emergence $= v = ?$

Rate of water emergence $= R = ?$

Calculations :-

(a). According to Torricelli's theorem

$$v = \sqrt{2g\Delta h}$$

$$= \sqrt{2 \times 9.8 \times 15} \text{ ms}^{-1}$$

$$\boxed{v = 17.14 \text{ ms}^{-1}}$$ Answer

(b). From equation of continuity

Volume flow rate $= Av$

$v = 17.14\ ms^{-1} = 1714\ cms^{-1}$

Volume flow rate $= 1714\ cms^{-1} \times 0.06\ cm^2$

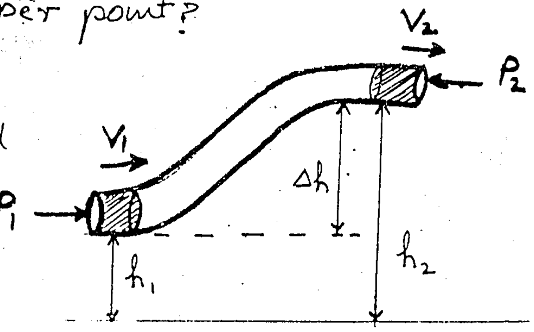
$= 102\ cm^3 s^{-1}$

So volume of water flows out in one second

$\boxed{V = 102\ cm^3}$ Answer

P. 6.4 :- Water is flowing smoothly through a closed pipe system. At one point the speed of water is $3\ ms^{-1}$, while at another point 3 m higher, the speed is $4.0\ ms^{-1}$. If the pressure is 80 kPa at the lower point, what is pressure at the upper point?

Solution :-



Data :-

Speed of water on lower end $= V_1 = 3\ ms^{-1}$

speed of water on upper end $V_2 = 4\ ms^{-1}$.

pressure at lower end $= P_1$

$= 80\ kPa = 80 \times 10^3\ Pa = 80 \times 10^3\ Nm^{-2}$

pressure at upper end $P_2 = ?$

Height of water $\Delta h = h_2 - h_1 = h_1 - h_2 = 3\ m$

Calculations :-

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

or

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2)$$

$$= 80000 + \frac{1}{2} \times 10^3 (3^2 - 4^2) + (1000 \times 9.8 \times 3)$$

$$= 47100\ Pa$$

$$= 47.\cancel{1}00 \times 10^3\ Pa$$

$\boxed{P_2 = 47\ kPa}$ Answer

P. 6.5 :- An aeroplane wing is designed so that when the speed of the air across the top of the wing is $450\ ms^{-1}$, the speed of air below the wing is $410\ ms^{-1}$. What is the pressure difference between the top and bottom of the wings?

SOLUTION :-

DATA :-

Speed of air on top $= V_1 = 450\ ms^{-1}$
Speed of air below $= V_2 = 410\ ms^{-1}$
Density of air $= \rho = 1.29\ kg\,m^{-3}$
Difference in pressure $= \Delta P = ?$

CALCULATION :-

According to Venturi relation.

$$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

$$\Delta P = \frac{1}{2}\rho(V_1^2 - V_2^2)$$

$$= \frac{1}{2}\times 1.29\{(450)^2 - (410)^2\}$$

$$= \frac{1}{2}\times 1.29\times 34400$$

$$= 22188\ Pa$$

$$\Delta P = 22.188\times 10^{3}\ Pa$$

$$\boxed{\Delta P = 22\ KPa}$$ Answer

P. 6.6 :- The radius of the aorta is about 1.0 cm and the blood flowing through it has a speed of about $30\ cm\,s^{-1}$. Calculate the average speed of the blood in the capillaries using the fact that although each capillary has a diameter of about 8×10^{-4} cm, there are literally millions of them so that their total cross section is about $2000\ cm^2$.

Data :- Radius of aorta $= r_1 = 1\ cm$
Speed of blood $= V_1 = 30\ cms^{-1}$
Diameter of capillary $= d = 8 \times 10^{-4}\ cm$
Radius of " $= r = 4 \times 10^{-4}\ cm$
Total area of cross section of capillaries
$= A_2 = 2000\ cm^2$
Average speed of blood $= V_2 = ?$

CALCULATIONS :-

Area of aorta $= A_1 = \pi r_1^2$
$= 3.14 \times (1\ cm)^2$
$= 3\ cm^2$

From equation of continuity

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1}{A_2} V_1$$

$$= \frac{3.14\ cm^2}{2000\ cm^2} \times 30\ cms^{-1}$$

$$= 4.7 \times 10^{-2}\ cms^{-1}$$

$$4.7 \times 10^{-4}\ ms^{-1}$$

$\boxed{V_2 \simeq 5 \times 10^{-4}\ ms^{-1}}$ Answer

P. 6.7 :- How large must a heating duct be if air moving $3.0\ ms^{-1}$ along it can replenish the air in a room of $300\ m^3$ volume every 15 min? Assume the air's density remains constant.

SOLUTION :-

Data :-
Speed of air $= v = 3\ ms^{-1}$
Volume of air $= V = 300\ m^3$
Time $= t = 15\ min = 15 \times 60 = 900\ sec$
Size of duct $= r = ?$

CALCULATIONS :.

According to Equation of continuity

$$Av = \text{Volume flow rate}$$

$$Av = \frac{V}{t}$$

$$\pi r^2 v = \frac{V}{t} \qquad (\because A = \pi r^2)$$

$$r^2 = \frac{V}{\pi v t}$$

$$r = \sqrt{\frac{V}{\pi v t}} = \sqrt{\frac{300\ m^3}{3.14 \times 3\ ms^{-1} \times 900\ s}}$$

$$\boxed{r = 0.19\ m = 19\ cm}$$

P. 6.8 :. An airplane design calls for a "lift due to the net force of the moving air on the wing of about 1000 Nm^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the speed of flow past the lower wing surface is 160 ms^{-1} what is the required speed over the upper surface to give a lift of 1000 $N\ m^{-2}$? The density of air is 1.29 $kg\ m^{-3}$ and assume max. thickness of wing be one meter

SOLUTION :.

Data :- Pressure on wing $= \Delta P = 1000\ Nm^{-2}$
Speed of air $= V_1 = 160\ ms^{-1}$
Density of air $= \rho = 1.29\ kgm^{-3}$
Thickness of wing $=$ 1m
Speed of uplift $= V_2 = ?$

Calculation :-

According to Venturi relation

$$P_1 - P_2 = \frac{1}{2}\rho\,(V_2^2 - V_1^2)$$

$$\Delta P = \frac{1}{2}\rho\,(V_2^2 - V_1^2)$$

$$V_2 = \frac{2\Delta P}{\rho} + V_1^2$$

$$V_2^2 = \frac{2\times 1000\ Nm^{-2}}{1.29\ kg\,m^{-3}} + (160\ ms^{-1})^2$$

$$V_2 = \sqrt{\frac{2000}{1.29}\ m^2s^{-2} + (160)^2\ m^2s^{-2}}$$

$$V_2 = 164.77\ ms^{-1}$$

$$\boxed{V_2 = 165\ ms^{-1}}$$ Answer

P. 6.9 :- What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15m ?-

SOLUTION :-

Data :-

$\Delta h = 15m$

$g = 9.8ms^{-2}$

Density of water $\rho = 1000\ kgm^{-3}$

$\Delta P = ?$

Calculations :-

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

or

$$P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2}\rho (V_2^2 - V_1^2)$$

As speed of water throughout the flow does not change so $V_1 = V_2 \Rightarrow V_2^2 - V_1^2 = 0$

So

$$\Delta P = \rho g (h_2 - h_1) = \rho g \Delta h$$

$$\Delta P = 1000 \times 9.8 \times 15\ Nm^{-2}$$

$$\boxed{\Delta P = 1.47\times 10^5\ Pa}$$ Answer